An Analytical Solution to Reasonable Royalty Rate Calculations[a]

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Abstract

The courts are increasingly encouraging use of more rigorous, scientific approaches to royalty rate calculations. The technique proposed in this study applies a classic, peer-reviewed game theoretic model that yields an efficient and fair result. The model can be used to supplement the *Georgia-Pacific* template for a reasonable royalty rate calculation. This should allow patent infringement litigation to build on *Georgia-Pacific* by interpreting evidence and data in ways that reflect economic conditions governing the outcome of a hypothetical negotiation.

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[a] This paper reflects the opinions of the authors and not those of InteCap, Inc. The concepts and theories covered by this presentation are not intended to be all-inclusive on the topic of reasonable royalties. The concepts are for illustrative purposes and may not represent approaches that the authors or InteCap would recommend in a particular matter. The reader should keep in mind that each case should be evaluated in light of its own facts and circumstances.

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I. Introduction

The determination of a reasonable royalty rate to be found in a licensing agreement that doesn't exist and never existed is a formidable assignment for licensing experts and triers of fact. Since 1970, *Georgia-Pacific v. U.S. Plywood Corp*.[1] has served as the conventional template for calculating such royalty rates. *Georgia-Pacific* sets forth fifteen factors to be considered in the context of a "hypothetical negotiation" between a willing licensee and a willing licensor at the time of the infringement.

The *Georgia-Pacific* template has been criticized on grounds that use of these factors can produce a royalty rate unsupported by economic theory: licensing experts run down the list and identify some factors in support of "high" rather than "low" royalty rates, while other factors are thought to point in the opposite direction. What can result is an unsound calculation shrouded by "reliance" on *Georgia-Pacific*. In fact, courts appear to tire of attempting to apply these factors as a group, often finding them unhelpful. As Judge Glasser noted in *Gasser Chair Company, Inc. v. Infanti Chair Manufacturing Corp*.[2],

> It would be an affectation of research to cite the countless cases which simply reiterate the *Georgia-Pacific* factors to be considered in determining a reasonable royalty…To set out those fifteen factors would also needlessly burden this decision

The testimony of licensing experts can be strengthened by consideration of economic theory, rather than solely the identification of which factors in the *Georgia-*

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[1] *Georgia-Pacific Corp v. U.S. Plywood Corp.*, 318 F. Supp. 1116, 1121 (S.D.N.Y. 1970), modified, 446 F. 2d 295 (2d Cir. 1971), *cert. denied*, 404 U.S. 870, 92 S. Ct. 105, 30 L. Ed. 2d 114 (1971).

[2] *Gasser Chair Company, Inc. v. Infanti Chair Manufacturing Corp.*, 943 F. Supp. 201 (1996).

Pacific list support "high" rather than "low" royalty rates. We are not suggesting that the *Georgia-Pacific* factors be abandoned; they provide a good reference and starting point. Instead, we suggest that licensing experts also focus on two economic concepts that often are central: (1) anticipated profitability of the technology and (2) relative bargaining power of the participants. While the other *Georgia-Pacific* factors need not be ignored, these two areas should be closely examined when data permit.

Our attempt to narrow the focus on profitability and relative bargaining position is not novel to the discussion of reasonable royalty calculations. In *Honeywell v. Minolta*,[3] Judge Wolin replaced *Georgia-Pacific* factor number twelve with the anticipated profits and losses that the parties reasonably anticipated as a consequence of consummating a licensing agreement. Judge Wolin also cited relative bargaining position as an important factor. Furthermore, the first two *Georgia-Pacific* factors relating to established royalties and other comparable agreements were omitted from the *Honeywell* analysis. These additions in *Honeywell* have been described as helpful to licensing experts in the determination of "commercially realistic" royalties.[4]

We suggest that the two-person bargaining game as described by John Nash[5] accommodates the need for a clear and precise methodology that relies exclusively on anticipated profitability and relative bargaining power in the calculation of a reasonable royalty. The Nash Bargaining Solution ("NBS") has been called the most fundamental model in bargaining theory, which looks for a sharp prediction of the bargaining outcome

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[3] *Honeywell v. Minolta*, Civil Nos. 87-4847, 88-1624 (D.N.J. 1992).

[4] Robert Goldscheider, The Employment of Licensing Expertise in the Arena of Intellectual Property Litigation, 36 IDEA: The Journal of Law and Technology 159 (1996),

[5] John Nash, The Bargaining Problem, 18 Econometrica 155 (1950); John Nash, Two-Person Cooperative Games, 21 Econometrica 128 (1953).

based on the bargaining strengths of each side. The NBS is well supported by economic theory and is regarded as one of the simplest yet most fruitful paradigms in game theory.[6] The analytical clarity of the NBS also is an important justification for its use as another useful tool in calculating a reasonable royalty.

II. Nash Bargaining Solution

Nash obtained his solution by developing a set of reasonable conditions, or axioms, that any plausible solution must satisfy. These are as follows:

1. Pareto efficiency; that is, there should be no other feasible allocation that is (a) better than the solution for one negotiator and (b) not worse than the solution for the other negotiator.

2. Negotiators must collectively behave in a rational manner such that neither side gets less in the bargaining solution than could be obtained in disagreement.

3. The solution is independent of any numeric specification;[7] that is, if we change the way we measure the payoffs when we construct a two-person bargaining problem, then the solution corresponds to the same outcome.[8]

4. Eliminating alternatives other than the disagreement profits (opportunity costs from licensing) that would not have been chosen should not affect the solution.

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[6] Alvin Roth, Axiomatic Models of Bargaining (1979); Abhinay Muthoo, Bargaining Theory with Applications (1999).

[7] More formally, the solution is independent of any numeric risk-neutral utility specification.

[8] But keeping the numeric scales equivalent to the original ones

5. If the disagreement profits of the two parties are equal in the bargaining problem, then the solution also should treat them equally.

Using an ingenious mathematical argument, Nash demonstrated that satisfying these conditions defines a unique solution where the bargaining outcome simply rests on each negotiator's back-up alternative and the potential benefits of cooperation. That is, the NBS requires only knowledge or estimation of (1) the "disagreement" profits of both the licensee and licensor and (2) the total profits from a licensing agreement. Once these elements are determined, the NBS yields a unique and efficient compromise.

To solve for the NBS, we must first identify the disagreement profits for the patent holder, the disagreement profits for the infringer/licensee, and the total profit from licensing. We define d_1 as the disagreement payoff for the patent holder, which represents the profit the patent holder expects to receive if the negotiation fails. Likewise, we define d_2 as the disagreement payoff for the infringer. The exact functional form of these disagreement payoffs depends on specific assumptions about the two firms and economic conditions. The feasible payoff from licensing is represented by Π, which is the total profit from licensing. We also define the variables p_1 and p_2 as profit for the patent holder and infringer/licensee, respectively, from licensing.

Nash demonstrated that the only point that satisfies the conditions outlined above is the one obtained by solving the following constrained maximization problem:

$$\max_{p_1,p_2}(p_1 - d_1)(p_2 - d_2) \tag{1}$$

subject to the following conditions:

$$p_1 \geq d_1 \tag{2}$$

$$p_2 \geq d_2 \tag{3}$$

$$p_1 + p_2 \leq \Pi . \tag{4}$$

When transfer payments are permitted between the two agents, the bargaining problem can be fully characterized by three factors: (1) the disagreement payoff for the patent holder; (2) the disagreement payoff for the infringer/licensee; and (3) the total transferable wealth available to the two firms from licensing.[9] Thus, the conditions for the equilibrium payoffs are:

$$p_1^* - d_1 = p_2^* - d_2, \tag{5}$$

$$p_1^* + p_2^* = \Pi, \tag{6}$$

where p_i^* is the equilibrium payoff for firm i.

Solving equations (5) and (6) yields the NBS:

$$p_1^* = d_1 + \frac{1}{2}(\Pi - d_1 - d_2), \tag{7}$$

$$p_2^* = d_2 + \frac{1}{2}(\Pi - d_1 - d_2), \tag{8}$$

$$p_1^* + p_2^* = \Pi . \tag{9}$$

Equations (7) and (8) have the following interpretation: the entities bargain over the partition of total profits (Π); they first agree to give each other the payment that they respectively would obtain from not reaching agreement; then, they split the remaining profits equally. For each firm, the agreement payoff is greater when its own disagreement point is higher and its opponent's disagreement point is lower. Therefore, the relative bargaining power will depend on each side's respective outside opportunities.

[9] Roger Myerson, Game Theory: Analysis of Conflict (1991). Transfer payment is an important assumption that can guarantee the given scale factors in a game will also be the natural scale factors for the NBS. Risk neutrality is also an important assumption when

The fundamental insight of the NBS is that the alternatives to agreement that are available to each side limit how good a bargain the other partner can obtain. These alternatives set a lower limit on the share each side willingly will accept. Under the NBS, the two sides called upon to split a pie will divide the bargaining surplus—which is bounded by each bargainer's threat point or reservation price—down the middle, so that each has an equal share. The equal split-of-bargaining-surplus solution, although a theoretical construct, has an intuitive and normative appeal as a solution in the sense that it satisfies both issues of efficiency and fairness.

An alternative way of thinking about the NBS is in the framework of an implicit arbitrator who tries to distribute the gains from trade or, more generally, from cooperation in a manner that reflects fairly the bargaining strength of the two negotiators. Once each side's disagreement payoffs are determined, an arbitrator applies the NBS to obtain an efficient and fair solution. In the following section, we apply the NBS to the calculation of a reasonable royalty.

III. A Formal Analysis of a Reasonable Royalty

A reasonable royalty may be defined as the amount a person, desiring to manufacture, use, or sell a patented article as a business proposition, would be willing to pay as a royalty and yet be able to make a reasonable profit. Many possibilities exist that can affect the relative bargaining positions between a patent holder and licensee/infringer. Other things equal, if the patent holder has alternative licensees, it can threaten credibly to

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we use transfer payments; however, in the context of firms negotiating over an agreement, the assumption is plausible.

leave the bargaining table, other things equal, and this will allow it to obtain the better deal. Also, if there are few available substitute technologies, the licensee has fewer outside opportunities and will do relatively worse in the negotiation. We start with a simple case with a non-producing firm that owns a patent with no substitutes and only one licensee capable of producing the technology. We will later expand the model by introducing different assumptions about the firms to see how they affect the solution.

A. Case 1: One-Supplier World

The simplest case is that of a research and development firm (licensor) that is incapable of manufacturing any product embodying the invention. Such a firm can earn profits through licensing. Furthermore, we assume only one company (licensee/infringer) has the production capabilities to exploit the licensor's technology. How much the licensee pays in royalties can be determined by the NBS.

In this example, since the licensor earns nothing without the licensee, the licensor's bargaining position ultimately rests on the licensee's outside alternatives. If negotiations break down, the licensee remains able to earn profits equal to its opportunity cost. If negotiation is successful, the joint profit from licensing is equal to monopoly profit.

The set-up and solution of the NBS is straightforward. The licensor's disagreement payoff is zero:

$$d_1 = 0 . \tag{10}$$

The licensee's disagreement payoff, d_2, is equal to the licensee's opportunity cost, which is the return foregone from manufacturing the technology. Finally, the joint profits from licensing is equal to monopoly profit:

$$\Pi = P_m Q_m - C_2(Q_m), \tag{11}$$

where $C_2(\cdot)$ is the licensee's cost function and the subscript m refers to a monopoly. Applying equations (7) through (9), the NBS for a licensing agreement for the licensor and licensee, respectively, are:

$$p_1^* = \frac{P_m Q_m - C_2(Q_m) - d_2}{2}, \tag{12}$$

$$p_2^* = d_2 + \frac{P_m Q_m - C_2(Q_m) - d_2}{2}, \tag{13}$$

$$p_1^* + p_2^* = \Pi = P_m Q_m - C_2(Q_m). \tag{14}$$

To solve for the per-unit royalty, equations (12) and (13) can be rewritten as:

$$p_1^* = rQ_m, \tag{15}$$

$$p_2^* = P_m Q_m - C_2(Q_m) - rQ_m, \tag{16}$$

where r represents the per-unit royalty. Solving for r yields the following formula for a reasonable royalty:

$$r = \frac{1}{2}[P_m - AC_2] - \frac{d_2}{2Q_m}, \tag{17}$$

where AC_2 is the licensee's average total cost.

The first part of equation (17) stipulates that the royalty rate should be established at one-half of the difference between price and average total cost. Hence, the greater the mark-up of the patented technology, the greater the royalty rate. The second part of equation (17) demonstrates that the royalty rate will decrease with the licensee's opportunity cost. In other words, the more lucrative the licensee's next best alternative, the lower the royalty rate paid to the licensor.

B. Case 2: Two-Supplier World

An alternative patent infringement scenario is where two firms—the patent holder and the infringer—possess production capabilities. The patent holder, however, has not initiated production at the time of infringement. Under these conditions, a bargaining range for a reasonable royalty will not exist absent one of two broad conditions: (1) the licensee is able to serve markets that the patent holder is unable to access, and/or (2) the licensee produces at lower marginal cost. Without either of these conditions, there exists no incentive for the patent holder to license the technology.

These conditions exist in the real world if the inventor does not possess a comparative advantage in production or sales, i.e., when licensees have access to better distribution facilities, sales staff, or marketing resources. For the purposes of analyzing this case, we assume that 1) the licensee can produce at lower costs and 2) it is in the patent holder's interests to license the entire market and withdraw from production.

In this instance, the disagreement payoff for the patent holder is the profit it can earn as the high-cost, sole producer of its patented product. The patent holder's disagreement payoff is written as:

$$d_1 = P_1 Q_1 - C_1(Q_1). \tag{18}$$

where $C_1(\cdot)$ is the patent holder's cost function and P_1 and Q_1 are the profit-maximizing price and quantity for the patent holder absent the infringer. The disagreement payoff for the licensee is again d_2.

The joint profit from licensing is similar to the previous case:

$$\Pi = P_m Q_m - C_2(Q_m). \tag{19}$$

It is assumed that $\Pi > d_1$ and that $\frac{\partial C_2}{\partial Q} < \frac{\partial C_1}{\partial Q}$. The NBS payoff for the licensor and licensee, respectively, are:

$$p_1^* = d_1 + \frac{P_m Q_m - C_2(Q_m) - d_1 - d_2}{2} = rQ_m, \tag{20}$$

$$p_2^* = d_2 + \frac{P_m Q_m - C_2(Q_m) - d_1 - d_2}{2} = P_m Q_m - C_2(Q_m) - rQ_m, \tag{21}$$

$$p_1^* + p_2^* = \Pi = P_m Q_m - C_2(Q_m). \tag{22}$$

The reasonable royalty is equal to:

$$r = \frac{1}{2}[P_m - AC_2] + \frac{1}{2Q_m}[d_1 - d_2]. \tag{23}$$

Equation (23) provides the general framework for the calculation of the reasonable royalty. The first part of equation (23) is identical to the first part of equation (17), as the royalty rate increases with the mark-up of the patented technology. The second part of the equation factors in the relative bargaining positions. If both sides have equal disagreement payoffs, then the additional profits achieved from licensing are split equally. However, the royalty rate changes as differences in the relative disagreement points or bargaining positions change. As one side's outside opportunity improves, the terms of the licensing agreement become more favorable.

C. Alternative Cases

The solution obtained in equation (23) provides a clear and efficient method of determining a reasonable royalty. Furthermore, it is adaptable to various situations that may surround the hypothetical negotiation. For instance, if there exist viable and non-infringing substitutes to the patented product, then the elasticity of demand for the patented

product is larger, which lowers the market power and profitability associated with the patent. The existence of substitute goods in the marketplace reduces the difference between price and average total cost in equation (23), $[P_m - AC_2]$, which points towards a lower royalty rate. This implies that the lower the profitability of a patented technology, the less the patent holder can charge in the licensing agreement. The existence of substitute products also will have the effect of lowering d_1, which further lowers the royalty rate. This result is consistent with and quantifies the conclusion by Culbertson and Weinstein that a reasonable royalty "depends fundamentally upon the extent and nature of substitute products for the patented product."[10]

In conclusion, the NBS results in an intuitively appealing royalty rate that reflects the economic conditions of the licensing agreement. Through the basic analysis of the total potential profit and the disagreement payoffs, the methodology of the NBS provides a clear way of quantifying the fair value of the technology between the patent holder and the licensee.

IV. NBS and the DCF Method

The analysis thus far has focused on a static situation. Although this provides an intuition for the calculation of a reasonable royalty, it does not address the fact that the underlying value of a technology is based on the present value of future economic benefits. Factors that can limit these benefits include the market potential, the sensitivity of

[10] John Culbertson and Roy Weinstein, Product Substitutes and the Calculation of Patent Damages, 70 J. Patent and Trademark Office Society 705 (1988).

profits to production costs, the period of time over which the benefits will be enjoyed, and other economic factors.

The Discounted Cash Flow ("DCF") method is a popular choice for calculating future economic benefits. The objective is to discount into a present value the cash flow from the licensing agreement, and to also discount into present value the cash flows for the patent holder and infringer in the absence of an agreement. An advantage of using DCF is that direct comparisons can be made between total profits and opportunity costs because present values are measured in today's dollars. After obtaining these values from a DCF method, we can calculate the royalty using the NBS.

To apply the DCF method, we must first estimate net cash flows of the patented technology from an agreement. The DCF covers the interval from the point at which infringement began to the time of patent expiration. The determination of a royalty also should provide an amount that represents a fair return on the value of the intellectual property with respect to the amount of investment risk accepted. The investment risk should consider advancing technology, competing technology, and government regulations. Accordingly, we discount this cash flow stream using the weighted average cost of capital ("WACC"). The WACC includes a portion for an appropriate return on equity and a return that is sufficient to satisfy debt obligations. Typically, the Capital Asset Pricing Model can be used to derive an appropriate rate of return.

Applying the DCF to the NBS for reasonable royalty is straightforward, as equation (23) needs to be slightly modified to reflect future time periods and the appropriate risks specific to the firms and the patented technology:

$$r = \frac{1}{2}\left[\sum_{t=1}^{T}\frac{\left(P_{mt} - AC_{2t}\right)}{\left(1+d_m\right)^t} + \sum_{t=1}^{T}\frac{d_{1t}}{Q_{mt}\left(1+d_1\right)^t} - \sum_{t=1}^{T}\frac{d_{2t}}{Q_{mt}\left(1+d_2\right)^t}\right] \qquad (24)$$

The variable d_i is each firm's WACC and d_m reflects the risk associated with the patented technology itself. Equation (24) indicates that each firm's disagreement payoffs over time must be discounted by each firm's WACC.

The DCF method requires sufficient information about the estimated cash flows during the relevant period. Application of the model in practice requires that information be gathered from knowledgeable manufacturing, research, and marketing estimates as close to the time of infringement as feasible. Estimates of market size and realistic penetration also must be acquired. Additional information that would be helpful would include estimates of: (1) investment requirements for additional types and amounts of manufacturing facilities and (2) costs associated with designs and marketing must be estimated.

The principal problem with implementing the DCF analysis is the reliability of the data or estimates. From our experience, many internal financial projections, particularly those used to obtain financing, can be accepted as reasonable. Furthermore, discovery makes available internal marketing forecasts for both companies. Finally, the *Georgia-Pacific* factors can provide a financial framework.

We caution that not every projection can be taken seriously. Care must be taken in discerning when the projections were made and what methods were undertaken. Market analyst reports produced by investment banks may also provide market projections that can supplement these internal marketing forecasts.

V. Conclusion

The need for an objective and sound analysis of reasonable royalties in patent infringement litigation suggests consideration of adding this new technique to the use of the *Georgia-Pacific* factors. Data permitting, the seminal two-person bargaining game described by John Nash represents a peer-reviewed methodology that can be used to calculate a reasonable royalty from a hypothetical negotiation. The theoretical support for the NBS is overwhelming and, in the context of patent litigation, the reasonable royalty solution derived from the NBS is fair, efficient, and sensible.

The method of assigning weights to the *Georgia-Pacific* factors may produce a result that can be significantly improved and refined by the use of the NBS. Given the requirement that the parties conduct a "hypothetical negotiation" and agree to a hypothetical royalty rate, such a result is not surprising. By supplementing *Georgia-Pacific* through use of the NBS as the template for a reasonable royalty calculation, reasonable royalty experts may have a further tool to construct opinions regarding the profitability of the patented technology and the back-up alternatives of the parties in dispute. This technique thus may contribute to improving patent infringement litigation fact-finding and damages calculation.

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